A Generalized Formulation of Electromagnetically Coupled Striplines

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Abstract —**This paper presents a method of derivation of general expression for coupling which can be applied to coplanar as well as noncoplanar strips of equal and unequal widths arbitrarily located between ground planes and filled with layered substrate. Assuming TEM-mode propagation, the couping coefficient is determined in terms of self and mutual inductances. The method of derivation of these circuit parameters is also presented. The expressions for the fields required for analysis are obtained from Green's function formulation. Comparison of the numerical data with those obtained by other methods is presented.**

I. Introduction

A NUMBER of investigations on the estimation of coupling between the planar strips have been reported in the literature [1]–[5]. Cohn [1] formulated the coupling between two planar strips of equal width located symmetrically between parallel ground planes using conformal mapping technique. Bryant and Weiss [2] evaluated coupling between two strips of equal width placed above a ground plane. Shelton [3] and Mosko [4] evaluated the coupling between a pair of offset parallel-coupled strip-transmission lines. The methods of formulation for the different configuration of coupled lines are different. The formulas derived can be applied to strips of identical width only. It would, however, be worthwhile to find a generalized formulation from which the coupling for all the above cases, including that of strips of unequal width, can be found.

In the present paper, analysis is carried out to derive a general expression from which the coupling between offset parallel, as well as coplanar strips, arbitrarily located between two ground planes or above a ground plane can be determined. A generalized expression for coupling between coupled striplines propagating TEM-mode can be obtained in terms of coefficients of magnetic coupling $k_L(=L_m/\sqrt{L_1L_2})$ and capacitive coupling $k_c(=C_m/\sqrt{(C_1+C_m)(C_2+C_m)})$ [6]. The coupling coefficient can, therefore, be determined from an evaluation of the mutual parameters L_m, C_m, and primary constants L_1, C_1, L_2, and C_2 of the coupled lines. In the present work, L_1, C_1 and L_2, C_2 are determined from the Green's function formulation. An expression for the mutual inductance L_m for the general case of two noncoplanar strips embedded in a layered dielectric between ground planes is derived from the magnetic energy used in bringing the two coupled lines into interaction [7]. C_m is expressed in terms of L_m by equating the stored electric and magnetic energies for a propagating line. The general formulas for L_m and C_m so derived can be used to find the coupling between coplanar and noncoplanar strips of identical as well as dissimilar widths for arbitrary locations of these strips and upper ground plane. Comparison between the numerical results on coupling for the i) coupled strips of equal width located symmetrically between ground planes, ii) coupled strips of identical width above a ground plane, and iii) offset parallel coupled striplines, obtained from the present analysis, with those available in the literature is presented. Computation is also made for two strips of unequal widths located symmetrically between ground planes. The variation of coupling between two coplanar and noncoplanar strips as a function of position of the upper ground plane is also evaluated.

II. General Analysis

Consider the structure of Fig. 1(a) in which the region between the two parallel planes is filled with three dielectric layers having relative dielectric constants ϵ_1, ϵ_2, and ϵ_3 with the strip placed at the interface of the dielectrics having relative dielectric constants ϵ_2 and ϵ_3. Analysis is carried out for the case $h_1 \neq h_2 \neq h_3$. It is found that, for a point in the cross section of the stripline which is at a distance of 2.5 times the ground plane spacing in x direction, the transverse component of the electric field reduces to a negligibly small value [8]. For the purpose of analysis, therefore, the structure of Fig. 1(a) can be terminated by electric walls perpendicular to the ground planes. Since the objective of the analysis is to study the coupling between coplanar and noncoplanar strips, the electric walls perpendicular to the ground planes are assumed to be located asymmetrically with respect to the center of the strip. In the coordinate system shown in Fig. 1(b), the center of the strip is located at $x=((a/2)-S_1-(W_1/2))$.

The characteristic impedance of a TEM-mode line filled with multilayered dielectric can be determined from the expression

$$Z_{01}=\frac{1}{v_0\sqrt{C_1C_0}}=\frac{\sqrt{L_1}}{C_1} \tag{1}$$

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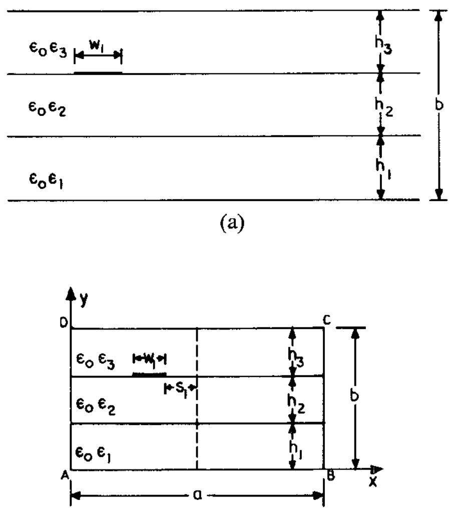

Fig. 1. (a) Planar strip between two ground planes filled with layered dielectric. (b) Configuration of (a) with finite ground planes terminated by electric walls on the two sides.

where v_0 is the velocity of light in free space, C_0 is the capacitance per unit length of the structure with air as dielectric, C_1 is the capacitance per unit length of the structure in the presence of layered dielectric, and L_1 is the inductance per unit length of the structure. The variational expression for the capacitance C_1 is given by [9]

$$\frac{1}{C_1}=\frac{\int\int \rho(x)\rho(x_0)G(x,y|x_0,h_1+h_2)\,dx\,dx_0}{\left[\int \rho(x)\,dx\right]^2} \tag{2}$$

where $\rho(x)$ is the charge distribution on the conducting strip, and $G(x,y|x_0,h_1+h_2)$ is the Green's function in the region occupied by the conductor. The potential distribution can be determined from the formula

$$\varnothing(x,y)=\int_{\text{strip}} G(x,y|x_0,h_1+h_2)\rho(x_0)\,dx_0. \tag{3}$$

The charge density at the edge of a thin strip approaches infinity. The mathematical expression for the corresponding charge distribution has been used by Joshi *et al.* [12] for the determination of field distribution in a microstrip line. Analysis based on this charge-distribution formula lead to a satisfactory agreement between theoretical and experimental results on the impedance of a microstrip-fed slot radiator. Using the same form of mathematical function as used by Joshi *et al.*, the expression for the charge distribution appearing in (2) can be assumed for the strip of Fig. 1(b) as

$$\rho(x)=\frac{\rho_0}{\sqrt{1-\left[\frac{2}{W_1}\left(x-\left(\frac{a}{2}-S_1-\frac{W_1}{2}\right)\right)\right]^2}},$$

$$\frac{a}{2}-S_1-W_1\leqslant x\leqslant\frac{a}{2}-S_1$$

$$=0 \qquad \text{otherwise.} \tag{4}$$

Using the appropriate boundary conditions, the Green's function required for the evaluation of potential function and capacitance can be obtained from the solution of the equation

$$\nabla^2 G=-\rho/\epsilon(\delta(x-x_0)) \tag{5}$$

where ϵ is the dielectric constant. The boundary and continuity conditions for the structure of Fig. 1(b) are given by [10]

$$\varnothing(x,0)=0 \tag{6a}$$

$$\varnothing(x,b)=0 \tag{6b}$$

$$\frac{\partial}{\partial y}\varnothing(0,y)=0 \tag{6c}$$

$$\frac{\partial}{\partial y}\varnothing(a,y)=0 \tag{6d}$$

$$\frac{\partial}{\partial x}\varnothing(x,h_1-0)=\frac{\partial}{\partial x}\varnothing(x,h_1+0) \tag{6e}$$

$$\frac{\partial}{\partial x}\varnothing(x,h_1+h_2-0)=\frac{\partial}{\partial x}\varnothing(x,h_1+h_2+0) \tag{6f}$$

$$\epsilon_0\epsilon_1\frac{\partial}{\partial y}\varnothing(x,h_1-0)=\epsilon_0\epsilon_2\frac{\partial}{\partial y}\varnothing(x,h_1+0) \tag{6g}$$

$$\epsilon_0\epsilon_2\frac{\partial}{\partial y}\varnothing(x,h_1+h_2-0)=\epsilon_0\epsilon_3\frac{\partial}{\partial y}\varnothing(x,h_1+h_2+0)-\rho(x,h_1+h_2). \tag{6h}$$

The Green's function $G(x,y|x_0,h_1+h_2)$ also satisfies the above boundary conditions when $\rho(x)=\delta(x-x_0)$. Using the appropriate boundary conditions for the Green's function, the solution of (5) is obtained as

$$G(x,y|x_0,h_1+h_2)=\sum_{n=1}^{\infty}\frac{2\epsilon_2}{n\pi\epsilon_0\Delta_n}\sinh\left(\frac{n\pi h_3}{a}\right)\sin\left(\frac{n\pi x_0}{a}\right)\sin\left(\frac{n\pi x}{a}\right)\cdot\sinh\left(\frac{n\pi y}{a}\right),\qquad 0\leqslant y\leqslant h_1 \tag{6i}$$

$$=\sum_{n=1}^{\infty}\frac{2}{n\pi\epsilon_0\Delta_n}\sinh\left(\frac{n\pi h_3}{a}\right)\left[\epsilon_1\cos\left(\frac{n\pi h_1}{a}\right)\cdot\sinh\left(\frac{n\pi}{a}(y-h_1)\right)+\epsilon_2\sinh\left(\frac{n\pi h_1}{a}\right)\cdot\cosh\left(\frac{n\pi}{a}(y-h_1)\right)\right]\cdot\sin\left(\frac{n\pi x_0}{a}\right)\sin\left(\frac{n\pi x}{a}\right),\qquad h_1\leqslant y\leqslant h_1+h_2 \tag{6j}$$

$$=\sum_{n=1}^{\infty}\frac{2}{n\pi\epsilon_0\Delta_n}\left[\epsilon_1\cosh\left(\frac{n\pi h_1}{a}\right)\sinh\left(\frac{n\pi h_2}{a}\right)+\epsilon_2\sinh\left(\frac{n\pi h_1}{a}\right)\cosh\left(\frac{n\pi h_2}{a}\right)\right]\sin\left(\frac{n\pi x_0}{a}\right)\cdot\sin\left(\frac{n\pi x}{a}\right)\sinh\left(\frac{n\pi}{a}(b-y)\right),\qquad h_1+h_2\leqslant y\leqslant b. \tag{6k}$$

Using (3), (4), (6i)–(6k), and integration formula (3.7532) of Gradshteyn and Ryzhik [15], the expression for the potential function in the different regions are obtained as

$$\varnothing_1(x, y) = \sum_{n=1}^{\infty} \frac{W_1 \pi \rho_0}{n\pi\epsilon_0 \Delta_n} J_0(X) \sin(Y) \epsilon_2 \sinh\left(\frac{n\pi h_3}{a}\right) \cdot \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi y}{a}\right), \quad 0 \leqslant y \leqslant h_1 \tag{7a}$$

$$= \sum_{n=1}^{\infty} \frac{W_1 \pi \rho_0}{n\pi\epsilon_0 \Delta_n} J_0(X) \sin(Y) \sin\left(\frac{n\pi x}{a}\right) \cdot \sinh\left(\frac{n\pi h_3}{a}\right) \cdot \left[\epsilon_1 \cosh\left(\frac{n\pi h_1}{a}\right) \sinh\left(\frac{n\pi}{a}(y-h_1)\right) + \epsilon_2 \sinh\left(\frac{n\pi h_1}{a}\right) \cdot \cosh\left(\frac{n\pi}{a}(y-h_1)\right)\right], \quad h_1 \leqslant y \leqslant h_1 + h_2 \tag{7b}$$

$$= \sum_{n=1}^{\infty} \frac{W_1 \pi \rho_0}{n\pi\epsilon_0 \Delta_n} J_0(X) \sin(Y) A \sin\left(\frac{n\pi x}{a}\right) \cdot \sinh\left(\frac{n\pi}{a}(b-y)\right), \quad h_1 + h_2 \leqslant y \leqslant b \tag{7c}$$

where $X = (n\pi W_1/2a)$, $y = (n\pi/a)((a/2) - (S_1 + (W_1/2)))$, and $J_0(X)$ is the Bessel function of the first kind of order zero

$$A = \epsilon_1 \cosh\left(\frac{n\pi h_1}{a}\right) \sinh\left(\frac{n\pi h_2}{a}\right) + \epsilon_2 \sinh\left(\frac{n\pi h_1}{a}\right) \cosh\left(\frac{n\pi h_2}{a}\right)$$

$$B = \epsilon_1 \cosh\left(\frac{n\pi h_1}{a}\right) \cosh\left(\frac{n\pi h_2}{a}\right) + \epsilon_2 \sinh\left(\frac{n\pi h_1}{a}\right) \sinh\left(\frac{n\pi h_2}{a}\right)$$

$$\Delta_n = B\epsilon_2 \sinh\left(\frac{n\pi h_3}{a}\right) + A\epsilon_3 \cosh\left(\frac{n\pi h_3}{a}\right).$$

Using (2), (4), and (7b), the capacitance of the structure of Fig. 1(b) is obtained in the form

$$\frac{1}{C_1} = \sum_{n=1}^{\infty} \frac{2}{n\pi\epsilon_0 \Delta_n} J_0^2(X) \sin^2(Y) A \sinh\left(\frac{n\pi h_3}{a}\right).$$

C_0 can be found from (8) with the substitution $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1$. Substituting C_1 and C_0 in (1), expressions for z_{01} and L_1 can be found.

Following exactly the same procedure and using the corresponding boundary conditions, it is found that, for a strip of width W_2 at the interface between the dielectrics having relative dielectric constants ϵ_1 and ϵ_2 with its center located at $x = ((a/2) + S_2 + (W_2/2))$, expressions for the potential function and capacitance are obtained as

$$\varnothing_2(x, y) = \sum_{n=1}^{\infty} \frac{W_2 \pi \rho_0}{n\pi\epsilon_0 \Delta_n} J_0(R) \sin(T) \cdot D \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi y}{a}\right), \quad 0 \leqslant y \leqslant h_1 \tag{9a}$$

$$= \sum_{n=1}^{\infty} \frac{W_2 \pi \rho_0}{n\pi\epsilon_0 \Delta_n} J_0(R) \sin(T) \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi h_1}{a}\right) \cdot \left[\epsilon_2 \sinh\left(\frac{n\pi h_3}{a}\right) \cosh\left(\frac{n\pi}{a}(h_1 + h_2 - y)\right) + \epsilon_3 \cdot \cosh\left(\frac{n\pi h_3}{a}\right) \sinh\left(\frac{n\pi}{a}(h_1 + h_2 - y)\right)\right], \quad h_1 \leqslant y \leqslant h_1 + h_2 \tag{9b}$$

$$= \sum_{n=1}^{\infty} \frac{W_2 \pi \rho_0}{n\pi\epsilon_0 \Delta_n} J_0(R) \sin(T) \epsilon_2 \sinh\left(\frac{n\pi h_1}{a}\right) \cdot \sin\left(\frac{n\pi x}{a}\right) \cdot \sinh\left(\frac{n\pi}{a}(b-y)\right), \quad h_1 + h_2 \leqslant y \leqslant b \tag{9c}$$

$$\frac{1}{C_2} = \sum_{n=1}^{\infty} \frac{2}{n\pi\epsilon_0 \Delta_n} J_0^2(R) \sin^2(T) D \sinh\left(\frac{n\pi h_1}{a}\right) \tag{10}$$

where

$$R = \frac{n\pi W_2}{2a}$$

$$T = \frac{n\pi}{a}\left(\frac{a}{2} + S_2 + \frac{W_2}{2}\right)$$

$$D = \epsilon_2 \cosh\left(\frac{n\pi h_2}{a}\right) \sinh\left(\frac{n\pi h_3}{a}\right) + \epsilon_3 \sinh\left(\frac{n\pi h_2}{a}\right) \cosh\left(\frac{n\pi h_3}{a}\right).$$

C_0 can be found from (10) with the substitution $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1$. Substituting C_2 and C_0 in (1), an expression for Z_{02} and L_2 can be found.

III. Expressions for Mutual Inductance and Coupling Coefficient

An expression for the mutual inductance is derived for the case of two planar strips located between parallel ground planes as shown in Fig. 2(a). As explained in Section II, the structure of Fig. 2(a) can be reduced to the one shown in Fig. 2(b), for the purpose of analysis. In Fig. 2(b), the sides AD and BC of the rectangle $ABCD$ can be assumed to be electric walls. Expressions for the mutual inductance L_m is derived from the relation [7]

$$L_m I_1 I_2 = \frac{1}{\mu_0} \int_v (\vec{B}_1 \cdot \vec{B}_2)\, dv \tag{11}$$

where I_1 and I_2 are the currents in the strips of width W_1 and W_2, respectively, and $\vec{B}_1$ and $\vec{B}_2$ are the flux densities in the region of interaction due to the strips

$$\vec{B}_1 = \mu \vec{H}_1 \tag{12a}$$

$$\vec{B}_2 = \mu \vec{H}_2 \tag{12b}$$

where $\vec{H}_1$ and $\vec{H}_2$ are the magnetic field intensities of the dominant mode fields.

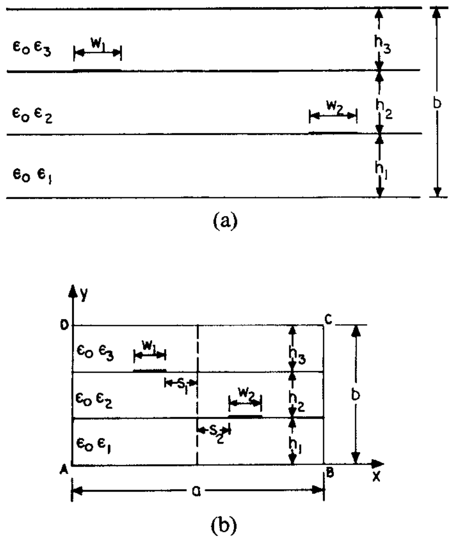

Fig. 2. (a) Offset parallel coupled strips between parallel planes filled with layered dielectric. (b) Configuration of (a) terminated by electric walls at finite distances from the strip.

If V_1 and V_2 are the applied voltages to the two coupled lines having strip widths W_1 and W_2, respectively, then

$$I_1 = V_1/Z_{01} \tag{13a}$$

$$I_2 = V_2/Z_{02}. \tag{13b}$$

The modal voltages for the dominant TEM-mode in the two lines are

$$V_1' = V_1\sqrt{\eta_1/Z_{01}} \tag{14a}$$

$$V_2' = V_2\sqrt{\eta_2/Z_{02}} \tag{14b}$$

where η_1 and η_2 are the wave impedances, and are given by $\eta_1 = 120\pi/\sqrt{\epsilon_{reff}} = \eta_2$. ϵ_{reff} is the effective dielectric constant. The modal currents for the magnetic fields are [11]

$$I_1' = V_1'/\eta_1 \tag{15a}$$

$$I_2' = V_2'/\eta_2. \tag{15b}$$

Since the magnetic field is equal to the product of the model current and the modal vector for the magnetic field, which is again related to the electric field [11], the expressions for $\vec{H}_1$ and $\vec{H}_2$ can be written as

$$\vec{H}_1 = I_1'(\vec{u}_z \times \vec{e}_1) \tag{16a}$$

$$\vec{H}_2 = I_2'(\vec{u}_z \times \vec{e}_2) \tag{16b}$$

where $\vec{e}_1$ and $\vec{e}_2$ are the modal vectors for the electric field and they satisfy the normalization conditions

$$\int\int e_1^2\, dS = \int\int e_2^2\, dS = 1. \tag{17}$$

The modal vectors $\vec{e}_1$ and $\vec{e}_2$ can be determined using the relation [12]

$$\vec{e}_1 = -\nabla_t \varnothing_1 \tag{18a}$$

$$\vec{e}_2 = -\nabla_t \varnothing_2. \tag{18b}$$

From (7a)–(7c), (9a)–(9c), and (11)–(18), and carrying on the integration over the region $X = (0 \text{ to } a), y = (0 \text{ to } h_1), (h_1 \text{ to } h_1 + h_2), (h_1 + h_2 \text{ to } b)$, an expression for mutual inductance per unit length, is obtained as

$$L_m = \frac{\mu_0}{R_1 R_2}\sqrt{\frac{Z_{01}Z_{02}}{\eta_1\eta_2}} \sum_{n=1}^{\infty} \frac{J_0(X)J_0(R)\sin(Y)\sin(T)}{n(\Delta_n)^2} \cdot \left[\frac{D\epsilon_2}{2}\sinh\left(\frac{n\pi h_2}{a}\right)\sinh\left(\frac{2n\pi h_1}{a}\right) + \sinh\left(\frac{n\pi h_1}{a}\right)\sinh\left(\frac{n\pi h_2}{a}\right) \cdot \sinh\left(\frac{n\pi h_3}{a}\right)\left(\epsilon_2^2 \sinh\left(\frac{n\pi h_1}{a}\right) \cdot \sinh\left(\frac{n\pi h_3}{a}\right) - \epsilon_1\epsilon_3\cosh\left(\frac{n\pi h_1}{a}\right)\cosh\left(\frac{n\pi h_3}{a}\right)\right) + \frac{A\epsilon_2}{2}\sinh\left(\frac{n\pi h_1}{a}\right)\sinh\left(\frac{2n\pi h_3}{a}\right)\right] \tag{19}$$

where

$$R_1^2 = \sum_{n=1}^{\infty} \frac{J_0^2(X)\sin^2(Y)}{n(\Delta_n)^2} \cdot \left[0.5\epsilon_2^2\sinh^2\left(\frac{n\pi h_3}{a}\right)\sinh\left(\frac{2n\pi h_1}{a}\right) + 0.5\sinh^2\left(\frac{n\pi h_3}{a}\right) \cdot \sinh\left(\frac{2n\pi h_2}{a}\right)\left(\epsilon_1^2\cosh^2\left(\frac{n\pi h_1}{a}\right) + \epsilon_2^2\sinh^2\left(\frac{n\pi h_1}{a}\right)\right) + \epsilon_1\epsilon_2\sinh\left(\frac{2n\pi h_1}{a}\right)\sinh^2\left(\frac{n\pi h_2}{a}\right)\sinh^2\left(\frac{n\pi h_3}{a}\right) + 0.5A^2\sinh\left(\frac{2n\pi h_3}{a}\right)\right]$$

$$R_2^2 = \sum_{n=1}^{\infty} \frac{J_0^2(R)\sin^2(T)}{n(\Delta_n)^2} \cdot \left[0.5D^2\sinh\left(\frac{2n\pi h_1}{a}\right) + 0.5\sinh^2\left(\frac{n\pi h_1}{a}\right) \cdot \sinh\left(\frac{2n\pi h_2}{a}\right)\left(\epsilon_3^2\cosh^2\left(\frac{n\pi h_3}{a}\right) + \epsilon_2^2\sinh^2\left(\frac{n\pi h_3}{a}\right)\right) + \epsilon_2\epsilon_3\sinh\left(\frac{2n\pi h_3}{a}\right)\sinh^2\left(\frac{n\pi h_1}{a}\right)\sinh^2\left(\frac{n\pi h_2}{a}\right) + 0.5\epsilon_2^2\sinh^2\left(\frac{n\pi h_1}{a}\right)\sinh\left(\frac{2n\pi h_3}{a}\right)\right]$$

μ_0 is the permeability of free space $= 4\pi \times 10^{-7}$ H/m.

For the transmission line supporting the dominant TEM-mode, the electric and magnetic energies are equal. The mutual inductance and capacitance satisfy the relation

$$L_m I_1 I_2 = C_m V_1 V_2. \tag{20}$$

Substituting (13a) and (13b) in (20)

$$C_m = L_m / Z_{01}Z_{02}. \tag{21}$$

The coefficients of electric and magnetic coupling are obtained from the formulas [6]

$$K_L = L_m/\sqrt{L_1 L_2} \tag{22a}$$

and

$$k_c = C_m/\sqrt{(C_1+C_m)(C_2+C_m)}\,. \tag{22b}$$

Substituting for C_m, C_1, and C_2 in (22b), an expression for the coefficient of electric coupling is obtained as

$$k_c = L_m\sqrt{\left(L_1+\frac{Z_{01}}{Z_{02}}L_m\right)\left(L_2+\frac{Z_{02}}{Z_{01}}L_m\right)}\,. \tag{23a}$$

For lines of identical characteristic impedance, the coefficient of electric coupling assumes the form

$$k_c = L_m/\sqrt{(L_1+L_m)(L_2+L_m)}\,. \tag{23b}$$

If L_m is negligibly small compared to L_1 and L_2, then

$$k_c = L_m/\sqrt{L_1 L_2}\,. \tag{23c}$$

Thus, for loosely coupled lines of identical characteristic impedance [13], $k_c = k_L$.

In the general case, the overall coupling coefficient is given by [6]

$$\bar{C} = \frac{k_L+k_c}{2}\left[\frac{\sin\theta_0}{\left(1-k_L k_c\cos^2\theta_0\right)^{1/2}}\right].$$

Maximum coupling is obtained when $\theta_0 = \pi/2$, i.e., when the coupled length is $\lambda/4$. Thus, for a coupled length of $\lambda/4$, the coupling coefficient is given by

$$\bar{C} = \frac{k_L+k_c}{2}. \tag{24}$$

IV. Evaluation of Coupling

The coupling has been formulated in terms of mutual inductance and mutual capacitance. The mutual inductance L_m is evaluated for the unfilled structure [6]. Numerical results are presented graphically for i) coplanar strips of equal and unequal width located symmetrically between ground planes, ii) coplanar strips of equal and unequal width located above a ground plane, iii) offset parallel coupled lines. The expressions for these quantities have appeared in the form of a series, each term of which contains Bessel function of the first kind of order zero. The computation is carried out using an HP-1000 computer. The standard subroutine for the Bessel function is utilized for the computation. For large values of the argument of the Bessel function, the assyoptotic expressions are used for the evaluation of the series. Coupling is evaluated for the following cases.

Case 1—Symmetric Strip

For $W_1/b = W_2/b = W/b$, $h_1/b = h_3/b = 0.5$, and $h_2 = 0$, the coplarar strips appear midway between ground planes. The resulting structure is a symmetric coupled stripline. The corresponding modified figure is shown as an inset in Fig. 3. Since the strip widths are identical, $L_1 = L_2 = L$. Using expressions (1), (8), (19), and (22a), L_1 and L_m of coplanar strips located midway between ground planes are evaluated for $W_1/b = W_2/b = W/b = 0.74$, $\epsilon_1 = \epsilon_2 = \epsilon_3 = 2.56$, $b/a = 0.2$, $h_2 = 0$, and $h_1/b = h_3/b = 0.5$. The

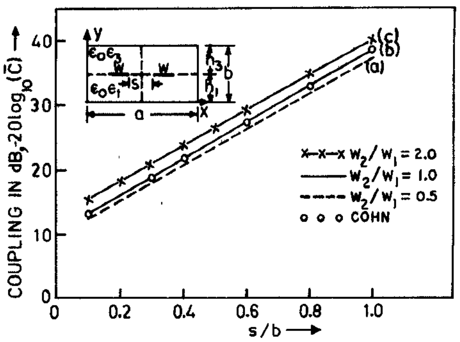

Fig. 3. Variation of coupling with S/b of two coplanar strips, symmetrically located between parallel planes for $\epsilon_1 = \epsilon_2 = \epsilon_3 = 2.56$. Curve a—$W_1/b = 0.74$, $W_2/b = 0.37$; curve b—$W_1/b = W_2/b = W/b = 0.74$; curve c—$W_1/b = 0.74$, $W_2/b = 1.48$.

numerical results reveal that, for the above range of the values of the parameters, L_m is negligibly small compared to L, and hence the results on coupling evaluated using (22a) or (23a) lead to almost identical results. The variation of coupling coefficient with $S/b(=(S_1+S_2)/b)$ is presented in Fig. 3 (curve b). For the sake of comparison, the corresponding results of Cohn are also presented in the same figure in the form of circles. Coupling is also evaluated for two coplanar strips of different widths located symmetrically between parallel planes. In this case, the coupling is calculated using (1), (8), (19), (22a), (23a), and (24) for $W_1/b = 0.74$ and $W_2/b = 1.48$ and the other parameters being the same as above. The variation of coupling as a function of S/b is presented in Fig. 3 (curve c). Curve a in the same figure presents results on coupling for $W_1/b = 0.74$ and $W_2/b = 0.37$.

Case 2—Microstripline

For $W_1/b = W_2/b = W/b$, $h_2 = 0$, and $h_3 \to \infty$, the resulting structure is a coupled microstripline. The corresponding modified figure is shown as an inset in Fig. 4. It is found from expressions (8), (10), and (19) that, when $h_2 = 0$ and the top ground plane is moved to infinity ($h_3 \to \infty$) and h_1 is kept fixed, $\epsilon_1 = \epsilon_2$ and $\epsilon_3 = 1.0$, the expressions for C_1, C_3, and L_m reduce to

$$\frac{1}{C_1} = \frac{2}{\pi\epsilon_0}\sum_{n=1}^{\infty}\frac{J_0^2(X)\sin^2(Y)}{n\left(\epsilon_1\coth\left(\frac{n\pi h_1}{a}\right)+1\right)} \tag{25a}$$

$$\frac{1}{C_2} = \frac{2}{\pi\epsilon_0}\sum_{n=1}^{\infty}\frac{J_0^2(R)\sin^2(T)}{n\left(\epsilon_1\coth\left(\frac{n\pi h_1}{a}\right)+1\right)} \tag{25b}$$

$$L_m = \frac{\mu_0}{R_{1m}R_{2m}}\sqrt{\frac{Z_{01}Z_{02}}{n_1 n_2}}\cdot\sum_{n=1}^{\infty}\frac{J_0(X)J_0(R)\sin(Y)\sin(T)\left(\coth\left(\frac{n\pi h_1}{a}\right)+1\right)}{n\left(\epsilon_1\coth\left(\frac{n\pi h_1}{a}\right)+1\right)^2} \tag{25c}$$

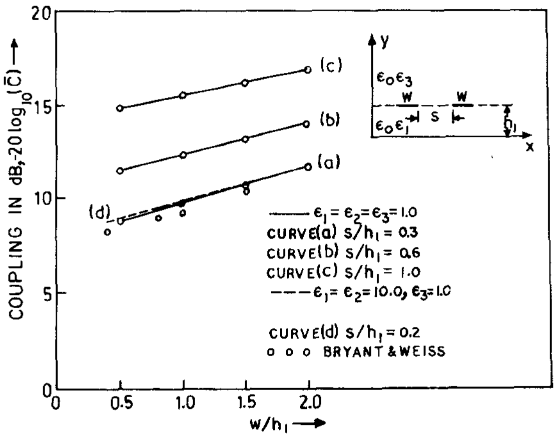

Fig. 4. Variation of coupling between two coplanar strips of equal width above a ground plane ($h_3 \to \infty$) as a function of W/h_1 with S/h_1 as a parameter for $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.0$ (curves a, b, c) and $\epsilon_1 = 10.0, S/h_1 = 0.2$ (curve d).

where

$$R_{1m}^2 = \sum_{n=1}^{\infty} \frac{J_0^2(X)\sin^2(Y)\left(\coth\left(\frac{n\pi h_1}{a}\right)+1\right)}{n\left(\epsilon_1 \coth\left(\frac{n\pi h_1}{a}\right)+1\right)^2}$$

$$R_{2m}^2 = \sum_{n=1}^{\infty} \frac{J_0^2(R)\sin^2(T)\left(\coth\left(\frac{n\pi h_1}{a}\right)+1\right)}{n\left(\epsilon_1 \coth\left(\frac{n\pi h_1}{a}\right)+1\right)^2}.$$

Corresponding values of characteristic impedance and inductance are obtained using (1).

The coupling coefficient is evaluated for two strips of equal width ($W_1 = W_2 = W$) above a ground plane as a function of W/h_1 with $S/h_1 (= (S_1 + S_2)/h_1) = 0.3$, 0.6, 1.0, and $\epsilon_1 = 1.0$. Using (1), (8), (22a), (23a), (25a), and (25c), the coupling coefficient is also evaluated for $\epsilon_1 = 10.0$ and $S/h_1 = 0.2$. The results are presented in Fig. 4. The data obtained by Bryant and Weiss are presented in the same figure in the form of circles.

Case 3—Offset Parallel Strip Transmission Lines (Fig. 2(b))

Using (1), (9), (19), and (22a), the coupling coefficient between two offset parallel strip transmission lines of equal width ($W_1 = W_2 = W$) is computed as a function of S/b with $h_1/b = h_2/b = h_3/b = 1/3$, $W/b = 0.6982$ (50-Ω line), $\epsilon_1 = \epsilon_2 = \epsilon_3 = 2.32$, and the results are presented in Fig. 5. The calculations are repeated for $h_1/b = h_3/b = 2/5$ and $h_2/b = 1/5$, $W/b = 0.7615$ (50-Ω line), $\epsilon_1 = \epsilon_2 = \epsilon_3 = 2.32$, and the results are presented in Fig. 5. For the sake of comparison, the corresponding results computed by Mosko [4] and Shelton [3] and available in the literature [14] for the loose coupling case are presented in the same figure.

Case 4—Coplanar Strips Asymmetrically Located Between Ground Planes (Fig. 2(b))

The formulation is used to compute the variation of coupling with change in position of the upper ground plane

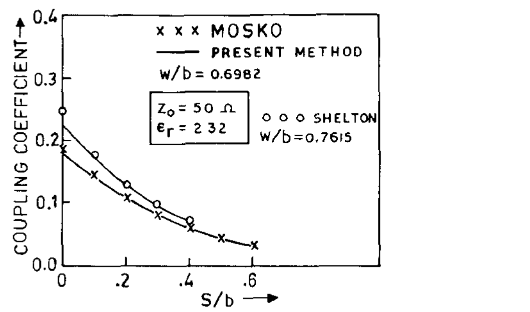

Fig. 5. Variation of coupling between two offset parallel strips of equal width between parallel planes as a function of S/b for $h_1/b = h_2/b = h_3/b = 1/3$ and $\epsilon_1 = \epsilon_2 = \epsilon_3 = 2.32$.

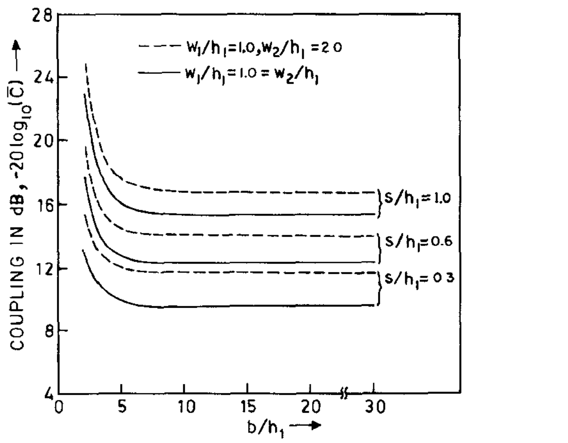

Fig. 6. Variation of coupling between two planar strips of equal and unequal widths as a function of b/h_1 with S/h_1 as a parameter.

for a fixed separation between the two coplanar strips and the lower ground plane. Using expressions (1), (8), (10), (19), (22a), (23a), and (24), the coupling coefficient is evaluated for $W/h = 1.0$, $h_1/a = 0.1$, and $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.0$ with $S/h_1 = 0.3, 0.6, 1.0$. The results are presented in Fig. 6. The coupling is also computed for $W_1/h_1 = 1.0$ and $W_2/h_1 = 2.0$ and the results are plotted in the same figure.

V. Conclusion

Agreement between the results obtained by the present method and those of Cohn for a symmetric stripline, Bryant and Weiss for a microstripline, and Mosko and Shelton for offset parallel strip transmission lines justifies the validity of the analysis. It is worthwhile to point out that, in the configuration used for the analysis, the value of characteristic impedance obtained by shifting the center of the strip away from the plane of symmetry up to a value of $S/b = 1.5$ when $a/b = 5.0$ has almost the same value as that for a strip between infinite ground planes. The formulation presented can, therefore, be used for estimation of coupling for all the structures considered. Most of the structures analyzed in the present work have also been analyzed earlier by other workers. Such structures are used for the design of directional detectors, power samplers and reflectometers, delay lines, bandpass filters, and impedance transformers. The method of analysis is, however, different for different structures. In the present work, one common formulation permits estimation of coupling for all the structures under consideration. The paper also furnishes additional information regarding estimation of coupling between strips of unequal width. Use of strips of unequal widths introduces an additional parameter in the adjustment of coupling. For all the configurations, expressions for the coupling have been obtained in the form of a series. The series has been found to converge and the contribution of terms beyond $n = 500$ is negligible.

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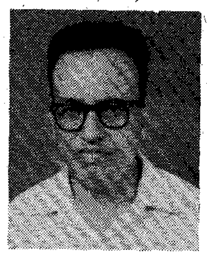

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